Nationwide Assessment of Potential Output from Wind-Powered Generators

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(Manuscript received 18 August 1975, in revised form 19 April 1976)

ABSTRACT

A method of computing power output from wind-powered generators has been developed and applied to estimate potential power output at various sites across the continental United States. The method assumes a wind-powered generator system which can be characterized by a cut-in speed V_0, a rated speed V_1 and a cut-out speed V_2. The generator output power is assumed to be constant at the rated power P_r between V_1 and V_2 and to vary parabolically from zero at V_0 to P_r at V_1. The wind distributions at various sites have been found to vary according to a Weibull distribution between realistic values of V_0 and V_1. Values of the Weibull distribution parameters at approximately 135 sites across the United States have been evaluated. These results have been projected to a constant height of 30.5 m (100 ft) and 61 m (200 ft) using data determined from observed Weibull parameter height variations at several meteorological tower sites across the country. A contour map is presented for generator capacity factor values (fraction of rated power output actually realizable). The capacity factor values were computed, using the above method, for wind-powered generator systems having cut-in speed $V_0 = 3.6$ m s^{-1} (8 mph), and rated speed $V_1 = 8.0$ m s^{-1} (18 mph), the characteristics of NASA's 100 kW Plumbrook unit, and $V_0 = 6.7$ m s^{-1} (15 mph), $V_1 = 13.4$ m s^{-1} (30 mph), hypothetical values for a 1 MW class unit. Results of the evaluation indicate that at a height of 61 m in the central United States and in certain portions of the New England coast over 60% of the rated output power can be achieved on an annual average basis, i.e., an average of $\geqslant 60$ kW from the Plumbrook 100 kW generator. In these same areas the 1 MW system would have over 20% capacity factors, i.e., an average of $\geqslant 200$ kW from the 1 MW system.

1. Introduction

Previous estimates of wind power potential in the United States (Thomas, 1945; Reed, 1974) have been based on statistics of the mean wind speed at various sites across the country. While the average power output of a wind-powered generator will certainly depend on the mean wind speed, it will also have a dependence on other factors, e.g., variance of wind speed about the mean. This report presents 1) a method for computing actual expected output power from a wind-powered generator, given the observed wind speed distribution, and 2) results of the application of this method, in the form of generator capacity factor (ratio of average power output to rated power) for the continental United States. Capacity factors are evaluated for wind-powered generators with characteristics of either NASA's 100 kW Plumbrook unit or a hypothetical 1 MW system.

2. Analysis method

For a known probability distribution $p(V)$ of wind speeds V, the average output power $\bar{P}$ from a wind-powered generator can be evaluated by

$$\bar{P} = \int_0^\infty P(V)p(V)dV, \qquad (1)$$

where $P(V)$ is the output power of the generator as a function of speed. The power function $P(V)$ for NASA's Plumbrook unit (Puthoff and Sirocky, 1974) is shown in Fig. 1. It can be described in terms of a cut-in speed V_0 of 3.6 m s^{-1} a rated speed V_1 of 8.0 m s^{-1} and a cut-out speed V_2 of 26.8 m s^{-1}. The rated power (actual generator electrical output) for the Plumbrook unit at rated speed is 100 kW. Analytically the power function $P(V)$ is described by

$$P(V) = \begin{cases} 0 & , \quad V \leqslant V_0 \\ A + BV + CV^2, & V_0 < V \leqslant V_1 \\ P_r & , \quad V_1 < V \leqslant V_2 \\ 0 & , \quad V > V_2 \end{cases} \qquad (2)$$

where V is the hub height wind speed, P_r the rated power, and A, B and C are coefficients determined by the following conditions:

$$\left. \begin{array}{l} A + BV_0 + CV_0^2 = 0 \\ A + BV_1 + CV_1^2 = P_r \\ A + BV_c + CV_c^2 = P_r(V_c/V_1)^3 \end{array} \right\}, \qquad (3)$$

where $V_c = (V_0 + V_1)/2$.

From a pragmatic viewpoint it is desirable to express the wind speed distribution as a two-parameter function of V. Two different two-parameter distributions, the
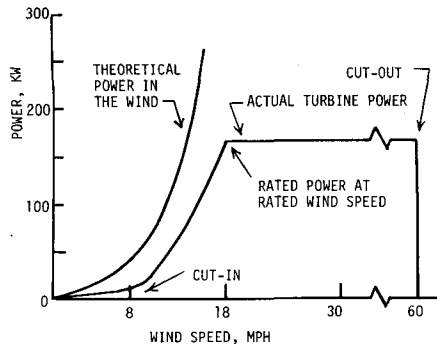
FIG. 1. Power output curve (Puthoff and Sirocky, 1974) used for the NASA 100 kW Plumbrook unit. Above 18 mph actual generator output is 100 kW.

Weibull and the log-normal, were selected for study. The bi-variate Gaussian was rejected because, in general, it would require specification of five parameters and hence would be too unwieldy. While theoretical problems are recognized with regard to the complete applicability of the log-normal or Weibull distribution at low wind speeds, these were considered as candidate distributions because 1) the analytical distribution need fit observed data only over the interval $V_0 \leqslant V \leqslant V_1$ [below V_0 there is no contribution to $\bar{P}$ from the integral in Eq. (1) and above V_1 the contribution to $\bar{P}$ is essentially the rated power times the cumulative probability of speeds above V_1], 2) both log-normal and Weibull parameters could easily be determined from observed wind speed frequency summaries by least squares techniques, and 3) the Weibull distribution of wind speed has found previous application in the study of wind loads on buildings (Davenport, 1963) and the log-normal distribution of wind speed has been used in air pollution studies (Luna and Church, 1974).

The Weibull distribution is expressed mathematically by

$$p(V)dV = (k/c)(V/c)^{k-1} \exp[-(V/c)^k]dV, \quad (4)$$

where c is the scale factor and k the shape factor. The scale factor c has units of speed and is closely related to the mean speed $\bar{V}$, since

$$\bar{V} = c\Gamma(1+1/k), \quad (5)$$

where Γ is the usual gamma function. For k in the range $1.4 < k < 3$, Eq. (5) shows that $c/\bar{V}$ values fall in the range $1.1 < c/\bar{V} < 1.3$. The shape factor k is dimensionless and inversely related to the variance σ^2 of wind speeds about the mean speed (i.e., high k values mean low variance and vice versa). Mathematically, this relationship is expressed by

$$\sigma^2 = c^2\{\Gamma(1+2/k) - [\Gamma(1+1/k)]^2\}. \quad (6)$$

Values of Weibull's c and k were determined from wind speed summary statistics (cumulative probabilities) for speeds within certain speed intervals. From integration of Eq. (4), the cumulative probability of finding a

speed less than V_x is given by

$$p(V \leqslant V_x) = 1 - \exp[-(V_x/c)^k], \quad (7)$$

which can be rearranged into the form

$$\ln\{\ln[1 - p(V \leqslant V_x)]\} = k \ln c - k \ln V_x, \quad (8)$$

from which k and c can be determined by least-squares fit of $y = a + bx$, where y is $\ln\{\ln[1 - p(V \leqslant V_x)]\}$, a is $k \ln c$, b is $-k$, and x is $\ln V_x$. If observed cumulative probabilities $p_{obs}(V \leqslant V_i)$ are available at a set of speeds V_i and the least-squares fit to Eq. (8) produces a set of calculated cumulative probabilities $p_{calc}(V \leqslant V_i)$, then the rms residual error ϵ given by

$$\epsilon^2 = \sum_i [p_{obs}(V \leqslant V_i) - p_{calc}(V \leqslant V_i)]^2 \quad (9)$$

can be used as a measure of "goodness-of-fit" of the distribution. Least-squares fit of both the Weibull and log-normal distributions showed that the Weibull distribution generally fits the observed distributions slightly better (i.e., produced smaller residual values ϵ). In many cases, however, a statistical test such as the Kolmogoroff-Smirnoff test showed that either distribution would adequately fit the observations. In view of the slightly smaller ϵ values generally produced by the Weibull distribution, this function was selected as preferable to the log-normal.

Weibull parameters c and k were determined at approximately 135 sites across the United States from wind summaries obtained from the National Climatic Center at Asheville, N. C. Only sites with constant anemometer heights during the period of the summary were used. Unfortunately, many sites had either changed anemometer heights during the period of the summary or had no readily available station history data for anemometer height (e.g., military sites). Only wind summaries representing 5 or more years of data were used, but any 5-year (or longer) periods after 1950 were considered acceptable.

From the Weibull distribution specified by the measured c and k values, the average power output $\bar{P}$ could be estimated for each site through numerical integration of

$$\bar{P} = \int_{V_0}^{V_1} (A + BV + CV^2)p(V)dV$$
$$+ P_r[p(V \leqslant V_2) - p(V \leqslant V_1)]. \quad (10)$$

This comes from substitution of the power function $P(V)$ from Eq. (2) into Eq. (1), with the distribution $p(V)$ in the integral of (10) being given by (4). Actual calculated average powers were expressed relative to rated power P_r by values of the capacity factor $F_c = \bar{P}/P_r$. For proper intercomparison between sites, capacity factors had to be converted to a common height level. The following section describes the height projection method developed.

3. Height variation of wind power

Meteorological tower wind summary data were obtained from four tower sites: Kennedy Space Flight Center, Florida; Wallops Island, Virginia; Hanford, Washington (Stone *et al.*, 1972); and WKY-TV in Oklahoma City, Oklahoma (Crawford and Hudson, 1970).

The Weibull scale factors c at all four tower sites were found to vary as a power law with height, i.e.,

$$c/c_0 = (z/z_0)^n, \qquad (11)$$

where the exponent n averaged 0.23 for all sites, with a standard deviation of 0.03. A logarithmic height variation for c would also have done well up to about 100 m, but the power law worked better when the entire height range (up to 400 m) was considered. The variation of c from site to site also averaged 0.03 (rms). Therefore, the terrain and other influences of the various sites affected c no greater than the seasonal influences. Seasonal influences on c probably arise from seasonal changes in prevailing wind direction (hence altering the effective terrain of the upwind fetch) and seasonal changes in distribution of atmospheric stability (which is known to influence the power law exponent). The average exponent for Eq. (3) of $n = 0.23$ is considered appropriate, therefore, for sites with roughness lengths varying from about 5 to about 50 cm (the range of values encompassed by the four tower sites studied).

The Weibull k values were found to vary in a manner which was similar for all tower sites, with a maximum near a height of 60 or 70 m. Each of these individual k variation curves was normalized as k/k_{max}, where
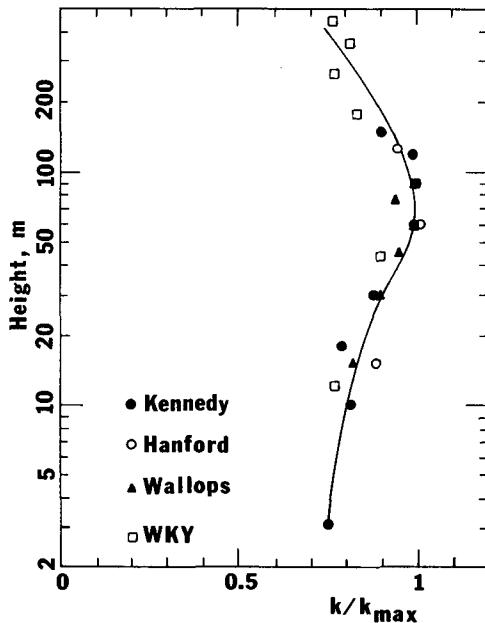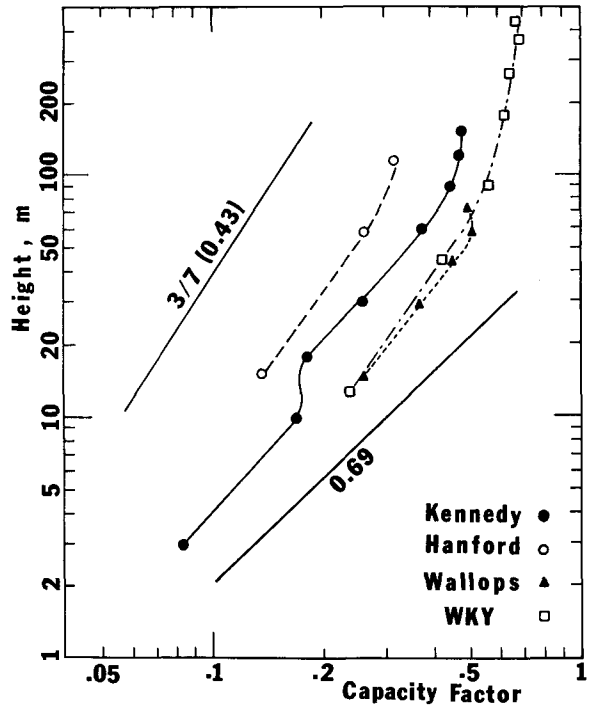

FIG. 3. Height variation of annual capacity factor at four tower sites. Plumbrook 100 kW power output characteristics ($V_0 = 3.6$ m s^{-1}, $V_1 = 8.0$ m s^{-1}) were used.

k_{max} was a subjectively determined maximum k value (at a height near 60 to 70 m). Since normalization by these k_{max} values produced completely similar curves for all tower data, a "universal" curve of k/k_{max} could be plotted. The power law for c and the "universal" k/k_{max} curve for k, shown in Fig. 2, were used to project c and k values measured at one height to any other desired height. Recently an improved method of height projection from one height ot another has been developed by Justus and Mikhail (1976). The newer method is based on a variable exponent n for Eq. (11), which depends on wind speed. For the wind power studies being reported here, however, the fixed exponent value of 0.23 was used.

Fig. 3 shows the observed height variation of the annual average capacity factor at the four tower locations. Capacity factors in Fig. 3 were evaluated for wind-powered generators with the characteristics of NASA's Plumbrook unit ($V_0 = 3.6$ m s^{-1}, $V_1 = 8.0$ m s^{-1}). Fig. 4 shows the same information on height variation of capacity factor for generators having operating characteristics of a hypothetical 1 MW unit ($V_0 = 6.7$ m s^{-1}, $V_1 = 13.4$ m s^{-1}). Comparison of Figs. 3 and 4 indicates that there is considerable influence of cut-in and rated speed on the height variation of generator power output, and that generators with different cut-in and rated speeds will have different optimum operating heights. Notice that the higher rated speed of the 1 MW unit allows it to make better use of increased winds with height than the 100 kW unit.
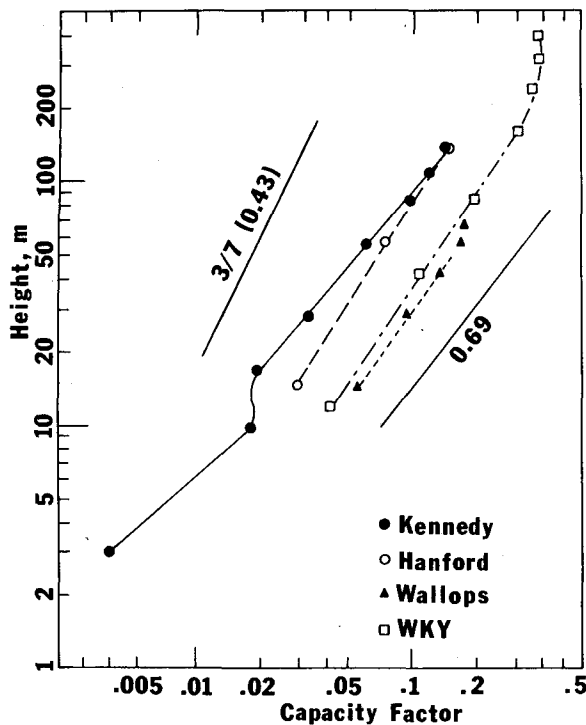

FIG. 2. Height variation of Weibull shape factor k, relative to maximum value k_{max} at about 60 m, for four tower data sets.

FIG. 4. Height variation of annual capacity factor using 1 MW design power output characteristics of $V_0=6.7$ m s^{-1}, $V_1=13.4$ m s^{-1}.

4. Wind power potential results

Figs. 5 and 6 show contour maps of annual average capacity factors for wind-powered generators operating at a height of 61 m and having characteristics of NASA's Plumbrook unit ($V_0=3.6$ m s^{-1}, $V_1=8.0$ m s^{-1}) and the hypothetical 1 MW unit ($V_0=6.7$ m s^{-1} and $V_1=13.4$ m s^{-1}). Both maps were evaluated for a height of 61 m and are considered to represent capacity factors above

relatively smooth terrain. The contours in these figures are accurate to about one contour interval in the higher capacity factor regions and one-half contour interval in the lower capacity factor regions.

From the results of Fig. 5 it can be concluded that wind-powered generators with operating characteristics near those of NASA's Plumbrook unit can be expected to operate, at the 61 m level, with capacity factors of better than 50% over substantial areas of the country. A capacity factor of 50% for a 100 kW unit would mean an average power output of 50 kW or an annual energy output of 4.4×10^5 kWh or 4.4×10^3 kWh per rated kW. At the same height level Fig. 6 shows that the hypothetical 1 MW unit can be expected to operate with capacity factors of 20% or better over substantial areas of the country. A plant factor of 20% or better for the 1 MW unit would mean an annual average power output of 200 kW or an annual energy output of 1.8×10^6 kWh or 1.8×10^3 kWh per rated kW.

Fig. 7 shows the capacity factor or specific output power (kWh per rated kW) as a function of annual average wind speed. Separate curves are given for the 100 kW unit with cut-in speed of 3.6 m s^{-1} and rated speed of 8.0 m s^{-1} and for the 1 MW unit with cut-in speed of 6.7 m s^{-1} and rated speed of 13.4 m s^{-1}. The curves in Fig. 7 were constructed from the calculated 30.5 m performance of the 100 kW and 1 MW systems at 135 sites. Fig. 7 shows an obvious trend of increasing output with increasing mean wind speed. The increase goes, however, not as the cube, but more like the square of the mean speed, especially for the 100 kW unit. However, there is considerable variation due to the variance of the wind, more so for the 1 MW unit than for the 100 kW unit, and it is possible to find pairs of sites for which the higher output occurs at the site with the lower mean speed (because of this influence of the variance). The results shown in Fig. 7 are very similar
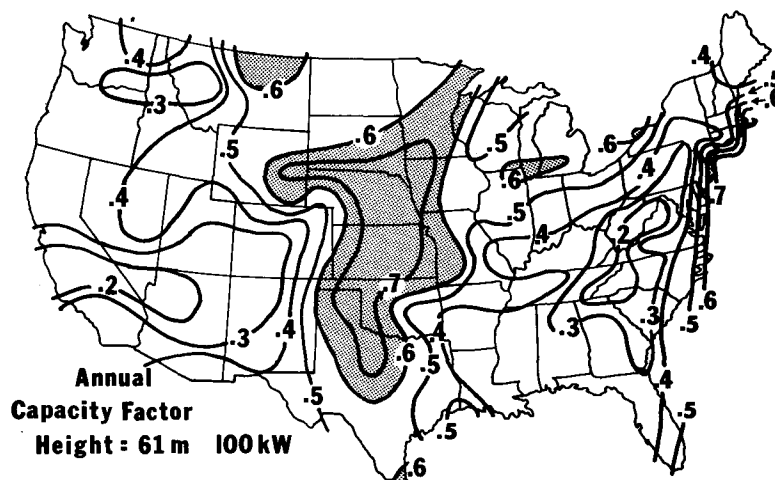

FIG. 5. Contours of annual capacity factor at a height of 61 m above flat terrain for generators with $V_0=3.6$ m s^{-1} and $V_1=8.0$ m s^{-1}.
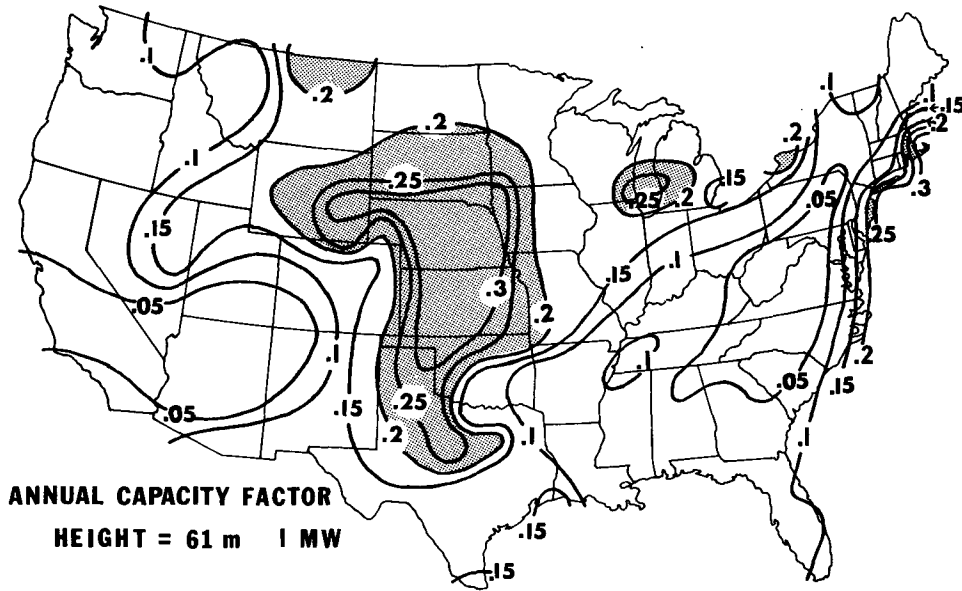
FIG. 6. As in Fig. 5 except for $V_0=6.7$ m s^{-1}, $V_1=13.4$ m s^{-1}.

to curves given in Fig. 57 of Golding (1956), except Golding did not indicate the strong dependence on variance of the wind speed. Dependence on variance is such that optimum system performance is found for low variance when average wind speeds are high (i.e., average wind is above rated speed and winds don't go below rated speed very often) and for high variance when average wind speeds are low (i.e., although the average wind is below rated speed, excursions of wind speed above rated speed occur at least some of the time).
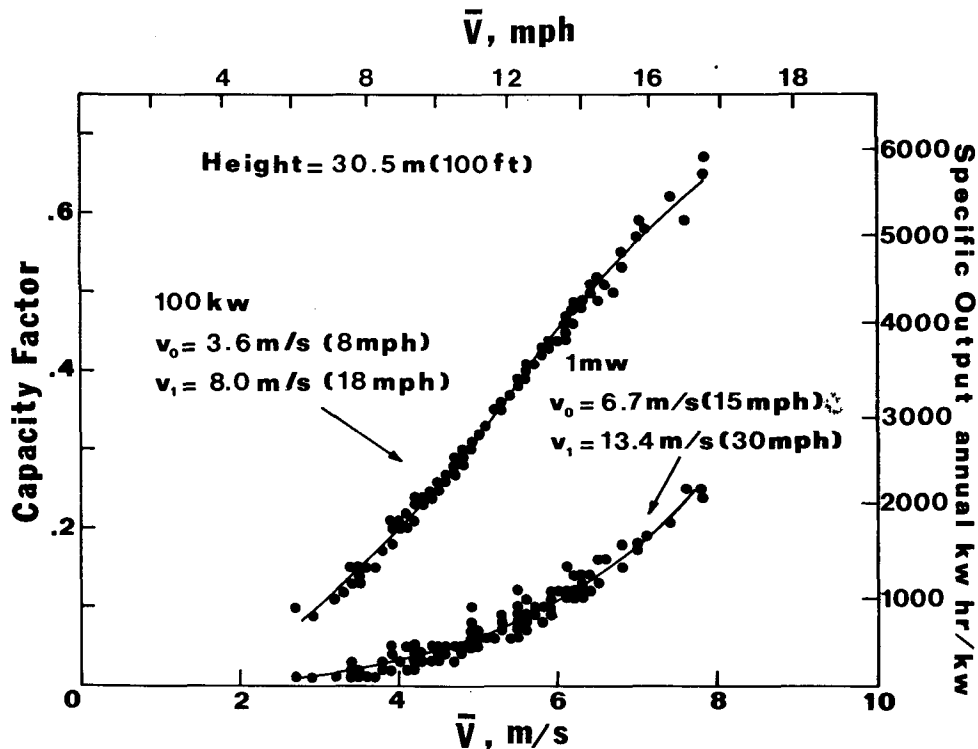

FIG. 7. Capacity factor or specific output power as a function of mean wind speed for 100 kW and 1 MW generators at simulated hub height of 30.5 m. Scatter of points is due to differences in wind variance (or Weibull k values) at 30.5 m height level.

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