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A joint probability density function of wind speed and direction for wind energy analysis

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Abstract

A very flexible joint probability density function of wind speed and direction is presented in this paper for use in wind energy analysis. A method that enables angular–linear distributions to be obtained with specified marginal distributions has been used for this purpose. For the marginal distribution of wind speed we use a singly truncated from below Normal–Weibull mixture distribution. The marginal distribution of wind direction comprises a finite mixture of von Mises distributions. The proposed model is applied in this paper to wind direction and wind speed hourly data recorded at several weather stations located in the Canary Islands (Spain). The suitability of the distributions is judged from the coefficient of determination R^2.

The conclusions reached are that the joint distribution proposed in this paper: (a) can represent unimodal, bimodal and bitangential wind speed frequency distributions, (b) takes into account the frequency of null winds, (c) represents the wind direction regimes in zones with several modes or prevailing wind directions, (d) takes into account the correlation between wind speeds and its directions. It can therefore be used in several tasks involved in the evaluation process of the wind resources available at a potential site. We also conclude that, in the case of the Canary Islands, the proposed model provides better fits in all the cases analysed than those obtained with the models used in the specialised literature on wind energy.

Keywords: Circular–linear distributions; Wind direction probability distributions; Wind speed probability distributions; Wind rose; Wind power density; Wind turbine energy output

1. Introduction

As pointed out by Koeppl [1], certain wind characteristics are important for the evaluation of wind resources and the design and performance of wind turbines. These include wind speed and direction probability distribution functions. The use of continuous wind speed probability density functions (pdf) is common [1–5]. However, less common is the use of the proposed model of continuous wind direction probability density functions [6–12].

In the evaluation of wind resources available at a given site of complex terrain or with several prevailing wind directions, it is interesting to have use of a joint probability density function. That is, to have use of a continuous model of the wind rose[1] that enables analysis of the variability of the energy characteristics of the wind in terms of speed and direction. Knowledge of these characteristics enables the wind turbines to be positioned in such a way as to maximise the capturable energy. It should be borne in mind that it could be that the most intense winds are not

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[1] The wind rose usually provides simultaneous information about the wind direction (normally for eight direction sectors) and intensity in graph or table form.

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those which blow most hours over the year from a particular direction. This is an important aspect to be taken into account when deciding on the orientation of the wind turbines.

The use of a joint probability density function of wind speed and direction is also useful in the modelling of the measure-correlate-predict method (MPC) [13]. However, the proposed use of continuous [9–12] and discrete [14] joint probability functions of wind speed and direction is scarce in the specialised literature on wind energy and other renewable energy sources.

The isotropic Gaussian model of McWilliams et al. [9,10] and Weber [11] was derived from the assumptions that the wind speed component along the prevailing wind direction (longitudinal component of the wind vector) is normally distributed with non-zero mean and a given variance, while the wind speed component along a direction orthogonal to that (lateral component of wind vector) is independent and normally distributed with zero mean and the same variance. The anisotropic Gaussian model of Weber [12] is a generalization of the model of McWilliams et al. [9]. In Weber [12], no restrictions are imposed on the standard deviations of the longitudinal and lateral fluctuations.

A very flexible joint probability density function of wind speed and direction is presented in this paper for wind energy analysis. A method proposed by Johnson and Wehrly [15] to obtain angular–linear distributions with specified marginal distributions has been used for this purpose. For the marginal distribution of wind speed we use a singly truncated from below Normal–Weibull mixture distribution, TNW-pdf, [4]. The marginal distribution of wind direction comprises a finite mixture of von Mises distributions [16]. The parameters of the model are estimated using the Least Squares method [17], which is resolved in this paper using the Levenberg–Marquardt algorithm [18]. The suitability of the distributions is judged from the coefficient of determination R^2 [18]. A comparison is made between the models presented by McWilliams et al. [9] and Weber [12] and the proposed joint distribution. This comparison is based on an analysis of the level of fit to the cumulative frequencies of hourly wind speeds and wind directions recorded at four weather stations located in the Canary Islands (Spain).

2. Models

2.1. Isotropic Gaussian model

The isotropic Gaussian model used by McWilliams et al. [9,10] and Weber [11] takes as its starting point the following hypotheses: (a) the existence of a prevailing wind direction; (b) the wind speed components for the prevailing wind direction (longitudinal, $v_{y'}$) and that which is perpendicular to it (lateral, $v_{x'}$) are random variables which are described by a Gaussian distribution; (c) the longitudinal and lateral components are statistically independent of each other; (d) the variances of the longitudinal, $\sigma_{y'}^2$, and lateral, $\sigma_{x'}^2$, components are the same; (e) the mean of the longitudinal component, $\mu_{y'}$, is other than zero, and the mean of the lateral component, $\mu_{x'}$, is null.

According to this model, the longitudinal component and the lateral component are described by the probability density functions (pdfs) given by Eqs. (1) and (2)

$$f_{y'}(v_{y'}) = \frac{1}{\sqrt{2\pi}\sigma_{y'}} \exp\left[-\frac{(v_{y'}-\mu_{y'})^2}{2\sigma_{y'}^2}\right] \tag{1}$$

$$f_{x'}(v_{x'}) = \frac{1}{\sqrt{2\pi}\sigma_{y'}} \exp\left[-\frac{(v_{x'})^2}{2\sigma_{y'}^2}\right] \tag{2}$$

In accordance with hypothesis (c), the joint pdf is given by the product of the two functions defined in Eqs. (1) and (2).

Performing a polar coordinate transformation, Eq. (3),[2] the joint pdf is obtained as a function of the speed and angle, Eq. (4)

$$v_{x'} = v\sin\theta'; \quad v_{y'} = v\cos\theta' \tag{3}$$

$$f_{V,\Theta}(v,\theta') = \frac{v}{2\pi\sigma_{y'}^2} \exp\left[-\frac{\mu_{y'}^2}{2\sigma_{y'}^2}\right] \exp\left[-\frac{v^2 - 2\mu_{y'}v\cos\theta'}{2\sigma_{y'}^2}\right] \tag{4}$$

The marginal pdf of the angle θ', Eq. (5), is obtained after integration over v in Eq. (4)

$$f_{\Theta}(\theta') = \frac{1}{2\pi}\exp\left(-\frac{\lambda^2}{2}\right)\left[1 - \sqrt{\pi}\xi\exp(\xi^2)\phi(\xi)\right]; \quad 0 < \theta' < 2\pi \tag{5}$$

where

$$\lambda = \frac{\mu_{y'}}{\sigma_{y'}}; \quad \xi = -\frac{\lambda\cos\theta'}{\sqrt{2}}; \quad \phi(\xi) = \frac{2}{\sqrt{\pi}}\int_{\xi}^{\infty}\exp(-t^2)\mathrm{d}t \tag{6}$$

The marginal pdf of the wind speed v, Eq. (7),[3] is obtained after integration over θ' in Eq. (4)

$$f_V(v) = \left[\frac{v}{\sigma_{y'}^2}\exp\left(-\frac{v^2}{2\sigma_{y'}^2}\right)\right]\exp\left(-\frac{\mu_{y'}^2}{2\sigma_{y'}^2}\right)I_0\left(v\frac{\mu_{y'}}{\sigma_{y'}^2}\right); \quad 0 < v < \infty \tag{7}$$

where $I_0(u)$ is the modified Bessel function of the first kind and order zero [19].

2.2. Anisotropic Gaussian model

The anisotropic Gaussian model proposed by Weber [12], uses as its starting point the same hypotheses as the isotropic model, with the exception of hypothesis (d). In the anisotropic model the two variances do not have to be the same.

[2] θ' represents the angles that the wind speeds form with the prevailing wind direction, y' (Fig. 1).

[3] The term in square brackets represents the Rayleigh probability density function, referred to in numerous wind energy papers [20].

According to this model, the longitudinal and lateral components are described by the probability density functions given by Eqs. (1) and (8)

$$f_{x'}(v_{x'}) = \frac{1}{\sqrt{2\pi}\sigma_{x'}} \exp\left[-\frac{(v_{x'})^2}{2\sigma_{x'}^2}\right] \quad (8)$$

In accordance with hypothesis (c), the joint pdf is given by the product of the two functions defined in Eqs. (1) and (8).

Performing a polar coordinate transformation, Eq. (3), the joint pdf is obtained as a function of the speed and angle that the wind speed forms with the axis of the prevailing wind direction, y', Eq. (9)

$$f_{V,\Theta}(v,\theta') = \frac{v}{2\pi\sigma_{y'}\sigma_{x'}} \times \exp\left[-\frac{(v\cos\theta' - \mu_{y'})^2}{2\sigma_{y'}^2} - \frac{v^2\sin^2\theta'}{2\sigma_{x'}^2}\right] \quad (9)$$

The marginal pdf of the angle θ', Eq. (10), is obtained after integration over v in Eq. (9)

$$f_\Theta(\theta') = \frac{1}{2\pi}\exp\left(-\frac{\lambda^2}{2}\right)\left[\frac{\gamma}{\gamma^2\sin^2\theta' + \cos^2\theta'}\right] \times \left[1 - \sqrt{\pi}\eta\exp(\eta^2)\phi(\eta)\right]; \quad 0 < \theta' < 2\pi \quad (10)$$

where

$$\gamma = \frac{\sigma_{y'}}{\sigma_{x'}}; \quad \eta = -\frac{\lambda\cos\theta'}{\sqrt{2\cos^2\theta' + 2\gamma^2\sin^2\theta'}}; \quad (11)$$

The marginal pdf of the wind speed v, Eq. (12), is obtained after integration over θ' in Eq. (9)

$$f_V(v) = \frac{v}{\sigma_{y'}\sigma_{x'}}\exp\left(-\frac{\lambda^2}{2}\right) I_0\left(v\frac{\mu_{y'}}{\sigma_{y'}^2}\right) \times \int_0^{2\pi}\exp\left[-\frac{v^2}{2\sigma_{y'}^2}(\cos^2\theta' + \gamma^2\sin^2\theta')\right]\mathrm{d}\theta'; \quad 0 < v < \infty \quad (12)$$

2.3. Proposed model

There are various methods of constructing joint distributions from information about the shape of the marginal distributions [21]. However, very little has been written in the scientific literature about angular–linear distribution models. In this paper, we have used the method proposed by Johnson and Wehrly [15] to obtain angular–linear distributions.

Johnson and Wehrly [15] define the probability density for an angular–linear distribution through Eq. (13)[4]

$$f_{V,\Theta}(v,\theta) = 2\pi g(\zeta) f_V(v) f_\Theta(\theta); \quad 0 \leqslant \theta < 2\pi; \ -\infty \leqslant v < \infty \quad (13)$$

where $g(\cdot)$ is the pdf of the circular variable ζ, given by Eq. (14)

$$\zeta = 2\pi[F_V(v) - F_\Theta(\theta)] \quad (14)$$

In the model proposed in this paper we use for wind speed pdf, $f_V(v)$, a Singly Truncated from below Normal Weibull mixture distribution, TNW-pdf. [4], Eq. (15). According to Carta and Ramírez [4], the mixture distribution proposed here provides very flexible models[5] for wind speed studies and can be applied in a widespread manner to represent the wind regimes in many regions. In addition, the TNW-pdf takes into account the frequency of null winds and, therefore, can represent wind regimes with high percentages of null wind speeds

$$f_V(v) = \frac{\omega_0}{I(\phi_1,\phi_2)} Z(v,\phi_1,\phi_2) + (1-\omega_0)\frac{\alpha}{\beta}\left(\frac{v}{\beta}\right)^{\alpha-1} \times \exp\left[-\left(\frac{v}{\beta}\right)^\alpha\right]; \quad 0 \leqslant v < \infty \quad (15)$$

where $Z(v,\phi_1,\phi_2)$ and $I(\phi_1,\phi_2)$ are given by Eq. (16)

$$Z(v,\phi_1,\phi_2) = \frac{1}{\sqrt{2\pi}}\exp\left[-\frac{(v-\phi_1)^2}{2\phi_2^2}\right]; \quad I(\phi_1,\phi_2) = \frac{1}{\phi_2}\int_0^\infty Z(v,\phi_1,\phi_2)\mathrm{d}v \quad (16)$$

where β is a scale parameter with the same units as the random variable and α is a shape parameter; ϕ_1 and ϕ_2 are parameters with the same units as the random variable; ω_0 is the weight in the mixture of the Singly Truncated from below Normal distribution ($0 \leqslant \omega_0 \leqslant 1$).

For the wind direction pdf, $f_\Theta(\theta)$, we use a mixture of von Mises distributions [16], Eq. (17). This mixture provides a very flexible model for wind direction studies and can be applied in a widespread manner to represent the wind direction regimes in zones with several modes or prevailing wind directions [16]

$$f_\Theta(\theta) = \sum_{j=1}^{N}\frac{\omega_j}{2\pi I_0(\kappa_j)}\exp[\kappa_j\cos(\theta-\mu_j)]; \quad 0 \leqslant \theta < 2\pi \quad (17)$$

In Eq. (17), N is the number of components of the mixture; $\kappa_j \geqslant 0$ and $0 \leqslant \mu_j < 2\pi$ are parameters; ω_j are nonnegative quantities that sum to one; that is, Eq. (18)

$$0 \leqslant \omega_j \leqslant 1, \ (j = 1,\ldots,N) \quad \text{and} \quad \sum_{j=1}^{N}\omega_j = 1 \quad (18)$$

The parameter μ_j is the mean direction and the parameter κ_j is known as the concentration parameter. Here, $I_0(\kappa_j)$

[4] θ represents the angles that the wind speeds form with the y axis direction.

[5] TNW-pdf can provide many good fits for unimodal, bimodal and bitangential wind speed frequency distributions. Bitangentiality occurs if there are two distinct points, v_1, v_2, at which there is a common tangent to the density curve. Thus, bitangentiality is implied by, but does not imply bimodality. Informally, bimodality implies an extra hump, but bitangentiality merely an extra bump.

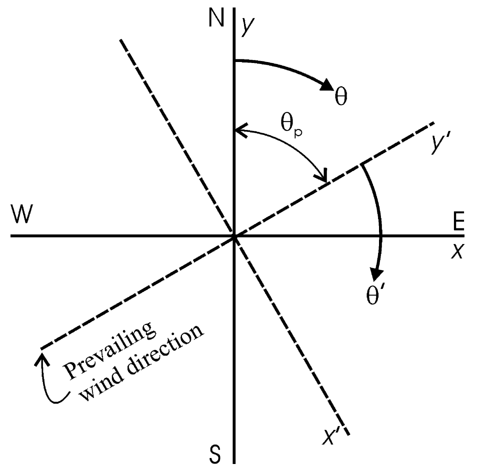

Fig. 1. Reference axes for the measurement of wind speed direction.

is the modified Bessel function of the first kind and order zero [19], and is given by Eq. (19)

$$I_0(\kappa_j) = \frac{1}{\sqrt{2\pi}} \int_0^{2\pi} \exp[\kappa_j \cos\theta]\mathrm{d}\theta = \sum_{k=0}^{\infty} \frac{1}{(k!)^2} \left(\frac{\kappa_j}{2}\right)^{2k} \tag{19}$$

The cumulative distribution functions for the speed, $F_V(v)$, and direction, $F_\Theta(\theta)$, of the wind are given by Eqs. (20) and (21), respectively

$$F_V(v) = \int_0^v f_V(v)\mathrm{d}v \tag{20}$$

$$F_\Theta(\theta) = \int_0^\theta f_\Theta(\theta)\mathrm{d}\theta \tag{21}$$

For the pdf of the circular variable ζ, in this paper we have used a mixture of von Mises distributions, Eq. (17).

3. Estimation of the parameters of the several models

3.1. Isotropic and anisotropic Gaussian models

Normally, in meteorology wind direction is measured in a clockwise direction[6] using the North as starting point (the y-axis in Fig. 1). However, in the models proposed by McWilliams et al. [9] and Weber [12], the angle measurement starting point is with respect to the prevailing wind direction (the y'-axis in Fig. 1). θ_p defines the position of the prevailing wind direction y' (and perpendicular x') in terms of the xy Cartesian system.

To estimate the parameters on which the isotropic and anisotropic models depend we will use the method of moments. In other words, the sample moments are equated to the corresponding moments in the population,[7] Eqs. (22) and (23)

$$\mu_{x'} = \bar{v}_{x'} = \frac{1}{n}\sum_{i=1}^{n}(v_{x'})_i = \frac{1}{n}\sum_{i=1}^{n} v_i \cos\theta'_i;$$
$$\mu_{y'} = \bar{v}_{y'} = \frac{1}{n}\sum_{i=1}^{n}(v_{y'})_i = \frac{1}{n}\sum_{i=1}^{n} v_i \sin\theta'_i \tag{22}$$

$$\sigma_{x'} = s_{x'} = \left\{\frac{1}{n-1}\sum_{i=1}^{n}[v_i\cos\theta'_i - \mu_{x'}]^2\right\}^{\frac{1}{2}};$$
$$\sigma_{y'} = s_{y'} = \left\{\frac{1}{n-1}\sum_{i=1}^{n}[v_i\sin\theta'_i - \mu_{x'}]^2\right\}^{\frac{1}{2}} \tag{23}$$

where n is the number of data of the sample and θ' is the wind direction with respect to the prevailing axis y'. If θ is the wind direction with respect to north of the xy Cartesian system, then we have, Eq. (24)

$$\theta' = \begin{cases} \theta - \theta_p & \text{if } \theta \geqslant \theta_p \\ 360 - \theta_p + \theta & \text{if } \theta < \theta_p \end{cases} \tag{24}$$

McWilliams and Sprevak [10] assume that the prevailing wind direction is given by the centre of the sector with the largest marginal frequency of occurrence. In this paper, θ_p is determined through Eq. (25) [16]

$$\theta_p = \begin{cases} \arctan\left(\frac{\bar{v}_y}{\bar{v}_x}\right); & \bar{v}_y \geqslant 0,\ \bar{v}_x > 0 \\ \frac{\pi}{2}; & \bar{v}_y > 0,\ \bar{v}_x = 0 \\ \pi + \arctan\left[\frac{\bar{v}_y}{\bar{v}_x}\right]; & \bar{v}_x < 0 \end{cases}$$
$$\theta_p = \begin{cases} \pi; & \bar{v}_y = 0,\ \bar{v}_x = -1 \\ 2\pi + \arctan\left[\frac{\bar{v}_y}{\bar{v}_x}\right]; & \bar{v}_y < 0,\ \bar{v}_x > 0 \\ \frac{3\pi}{2}; & \bar{v}_y < 0,\ \bar{v}_x = 0 \end{cases} \tag{25}$$

where $\bar{v}_x$ and $\bar{v}_y$ are the sample mean components with respect to the xy-axes.

3.2. Proposed model

As indicated in Section 2.3, the proposed model is built from marginal distributions of wind speed and wind direction. The parameters of these models are estimated in this paper using the Least Square (LS) method,[8] as described in Refs. [4,16].

From n sample wind speed and direction data, n values are calculated of the variable ζ, defined in Eq. (14), through the use of Eqs. (20), (21) and (26)

$$\zeta_i = 2\pi[F_V(v_i) - F_\Theta(\theta_i)]; \quad i = 1,\ldots,n \tag{26}$$

[6] In this paper the angle corresponding to the northerly direction is taken as angle 0°.

[7] This is the method used by McWilliams et al. [9] in an initial paper. McWilliams and Sprevak [10], in a second paper, used a method which has since been criticised [12,16].

[8] The Levenberg–Marquardt algorithm (LMA) [18] is used. The Mathcad? Software 2001i programme of MathSoft Engineering & Education, Inc., [22] is used to find the values of the parameters.

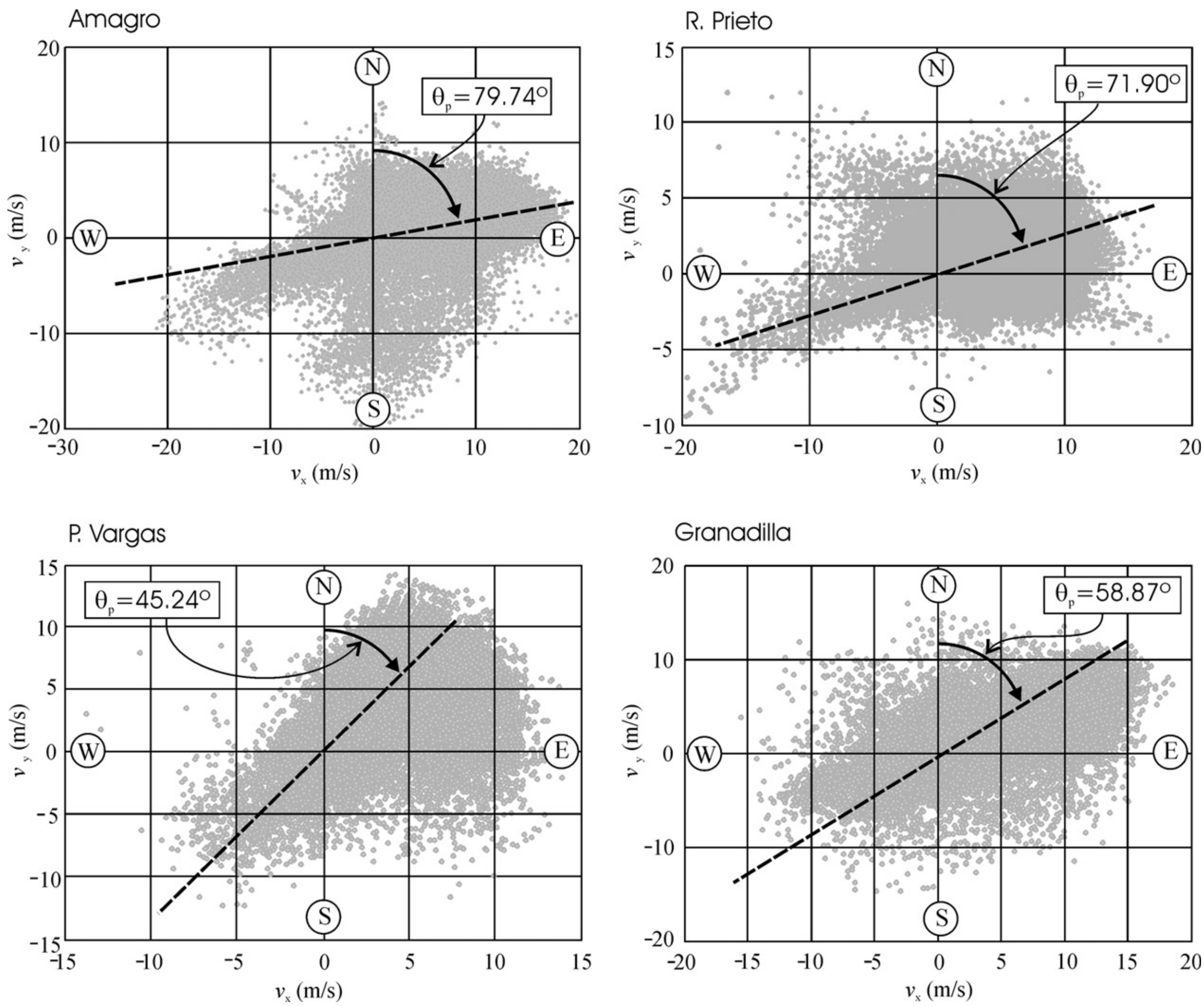

Fig. 2. Horizontal components, v_x and v_y, of the wind speed and estimated prevailing wind directions.

Those values of ζ obtained through Eq. (26) which are lower than zero are recalculated with Eq. (27), in such a way that $0 \leqslant \zeta_i < 2\pi$ for all $i = 1, \ldots, n$

$$\zeta_i = 2\pi + \zeta_i \text{ if } \zeta_i < 0; \quad i = 1, \ldots, n \tag{27}$$

In this paper, a mixture of von Mises distributions is fitted to this data sample of ζ and, following the procedure set out in Ref. [16], the parameters on which this mixture depends are determined.

4. Wind power density distribution and energy output of a wind turbine

An indicator of the size of the local wind energy resource is its annual mean wind power density [1,23,24]. Carta and Mentado [24] have proposed a model for wind power density and wind turbine energy output estimations. This model takes into account the time variability of air density ρ and wind speed, as well as the correlation existing between both variables.

However, in the literature related to wind energy, it is normally assumed that air density and wind speed are not correlated and is assumed that the air density is constant.[9] On this assumption, if we multiply the wind power density with the probability of each wind speed and wind direction, we have calculated the probability of wind power density at different wind speed and direction

$$P(v,\theta) = \frac{1}{2}\rho f_{V,\Theta}(v,\theta)v^3 \tag{28}$$

The output of a wind turbine, $E(v,\theta)$, with a power curve, PWT(v), in a wind regime with a joint probability density function, $f_{V,\Theta}(v,\theta)$, in a period t, can be expressed as a function of the wind speed and direction using[10]

[9] This value is usually 1.225 kg m^{-3}, corresponding to standard conditions (sea level, 15 °C). Carta and Mentado [24] recommend the use of the mean air density of the area when the variability of the air density and its correlation with the wind speed are not taken into account.

[10] The power curve of a wind turbine PWT(v), can be approximated by a piecewise linear function with a few nodes [25], or through cubic spline interpolation [24].

$$E(v,\theta) = t\int_{\theta_1}^{\theta_2}\left[\int_{v_1}^{v_2} PWT(v) f_{V,\Theta}(v,\theta)\mathrm{d}v\right]\mathrm{d}\theta \qquad (29)$$

5. Meteorological data used

In order to determine the flexibility of the proposed model to represent different wind regimes a study was carried out of samples recorded at different anemometer stations located in the Canarian Archipelago [4,26]. Four stations were selected which are representative of the most complex wind speed and direction distributions in the Canary Islands [4]. For the station called Amagro [4], mean hourly wind direction and wind speed data have been used from 7 years (1997–1999, 2001–2003, 2005). For the station called R. Prieto [4], data have been used from 4 years (1997–1998, 2001, 2005). For the station called P. Vargas [4], data have been used from 3 years (2002–2004). For the station called Granadilla [4], data have been used from 5 years (1998–2000, 2002, 2004). All the wind direction and wind speed records data were taken at a height of 10 m above ground level.

In Fig. 2 we can see the horizontal components, v_x and v_y, of the wind speed at the four stations under study, as well as the prevailing wind directions estimated using Eq. (25).

Table 1
Mean speeds, standard deviations and linear correlation coefficients of the wind speed for axes xy and $x'y'$

W. station	Cartesian axes xy					Axes $x'y'$				
	$\bar{v}_x$ (m s^{-1})	$\bar{v}_y$ (m s^{-1})	s_x (m s^{-1})	s_y (m s^{-1})	r_{xy} (–)	$\bar{v}_{x'}$ (m s^{-1})	$\bar{v}_{y'}$ (m s^{-1})	$s_{x'}$ (m s^{-1})	$s_{y'}$ (m s^{-1})	$r_{x'y'}$ (–)
Amagro	5.56	1.01	5.46	3.47	0.419	0	5.65	3.02	5.65	–0.229
R. Prieto	4.13	1.35	4.60	2.27	0.159	0	4.69	2.24	4.35	0.312
P. Vargas	3.51	3.48	3.43	3.83	0.403	0	4.94	2.82	4.30	–0.123
Granadilla	4.53	2.75	4.97	3.25	0.58	0	5.30	2.46	5.40	0.145

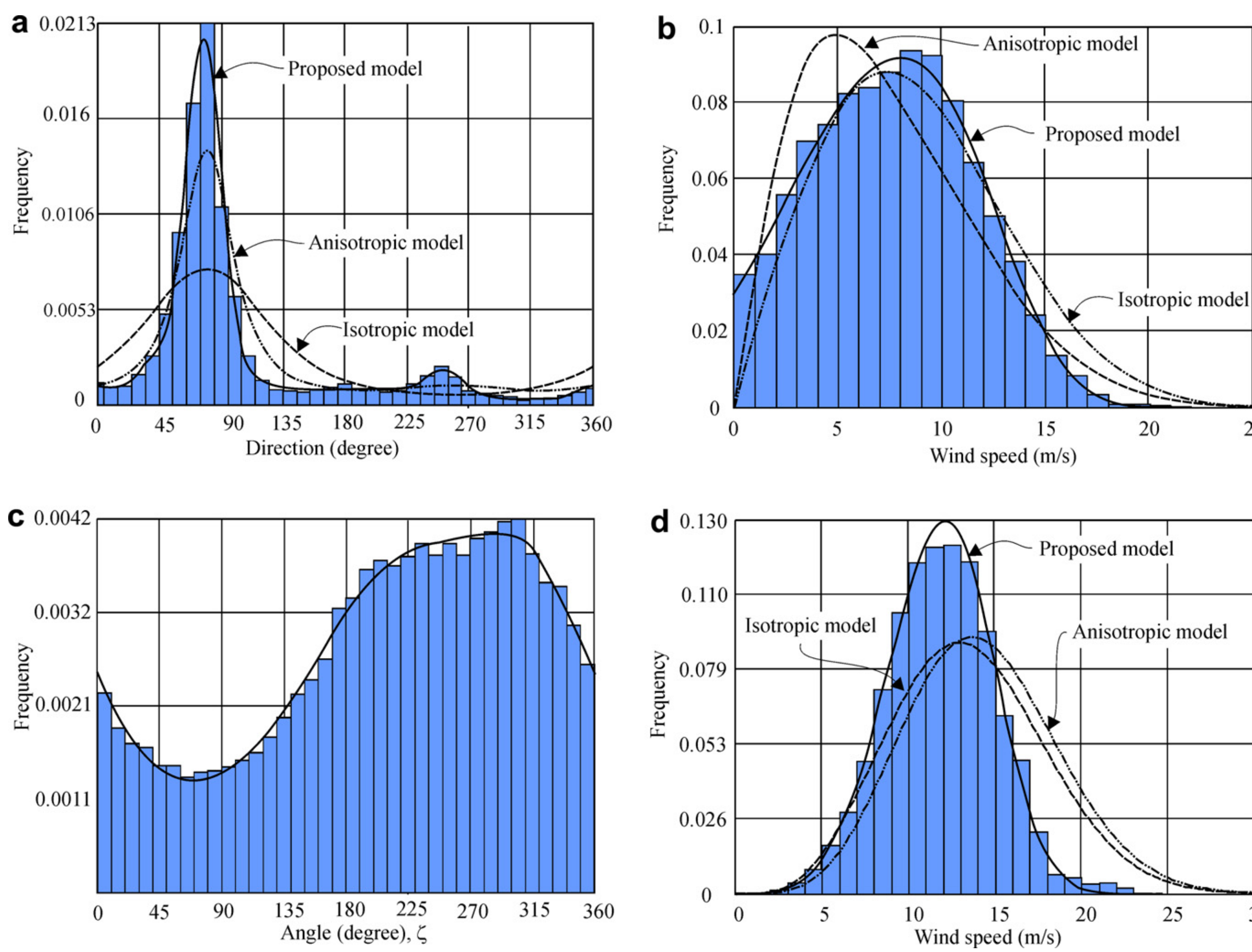

Fig. 3. Amagro station: (a) marginal distributions of wind direction, (b) wind speed and (d) wind power probability density function, for the three models analysed. (c) Probability density function of the variable ζ.

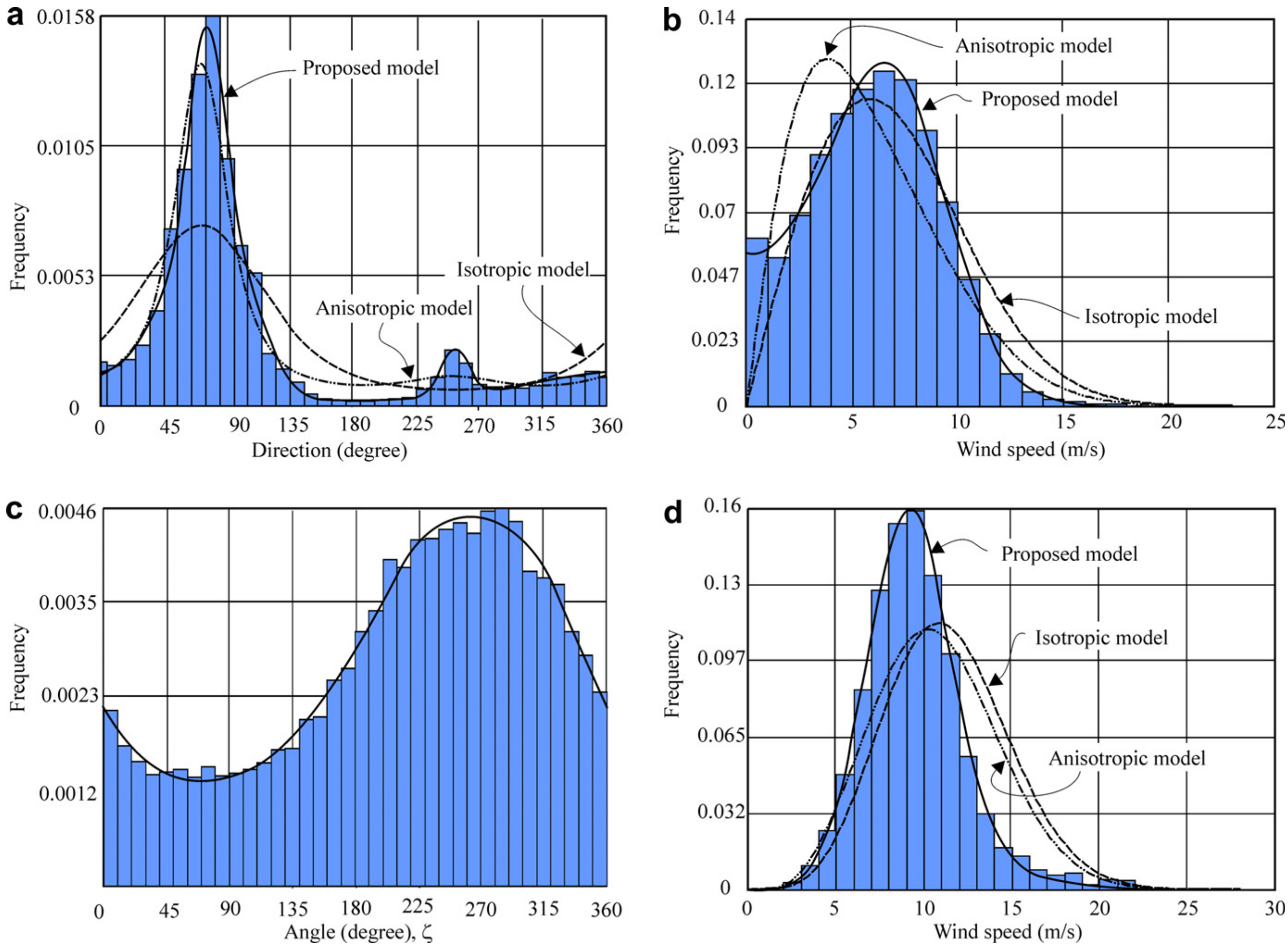

Fig. 4. R. Prieto station: (a) marginal distributions of wind direction, (b)wind speed and (d) wind power probability density function, for the three models analysed. (c) Probability density function of the variable ζ.

Table 1 shows the mean speeds, standard deviations and linear correlation coefficients, r_{xy} and $r_{x'y'}$, of the components of the wind speed for axes xy and $x'y'$, respectively.

6. Analysis of results

Fig. 3 shows, for the case of Amagro, the marginal distributions of the wind direction (Fig. 3a), wind speed (Fig. 3b), and wind power probability density functions (Fig. 3d) for the three models analysed. Also shown is the probability density function of the variable ζ (Fig. 3c), involved in the proposed model, Eqs. (13) and (14). The same graphs are represented in Figs. 4–6, but for the stations of R. Prieto, P. Vargas and Granadilla, respectively.

The parameters of the proposed marginal distributions of wind speed for the four stations under study are shown in Table 2. The proposed marginal distributions of wind speed have been built from a mixture of six von Mises distributions[11] and their parameters are shown in Table 3.

It can be seen in these figures that the proposed marginal distributions have a better degree of fit to the sample histograms than the marginal distributions obtained from the isotropic (Eqs. (5) and (7)) [9] and anisotropic models (Eqs. (10) and (12)) [12].

The probability density function $g(\zeta)$, Eq. (13), in which we see the existing relation between wind speed and direction has been represented by a mixture of two von Mises distributions (Table 4). In the cases analysed, the use of mixture distributions of more than two components has not provided an increase in the coefficient of determination, R^2, of the joint probability density functions. We should point out that when we have used the Maximum Likelihood method instead of the Least Squares method to estimate the parameters of the distribution, $g(\zeta)$, we have obtained a uniform distribution,[12] $g(\zeta) = 1/2\pi$. This indicates, as can be seen in Eq. (13), that according to the Maximum Likelihood method wind speed and wind direction are independent. However, we should point out that, in the cases analysed, the use of mixture distributions of

[11] The influence of the number of components of the von Mises mixture on the degree of fit is discussed in Ref. [16].

[12] When $\kappa = 0$, von Mises distribution is the uniform distribution [27].

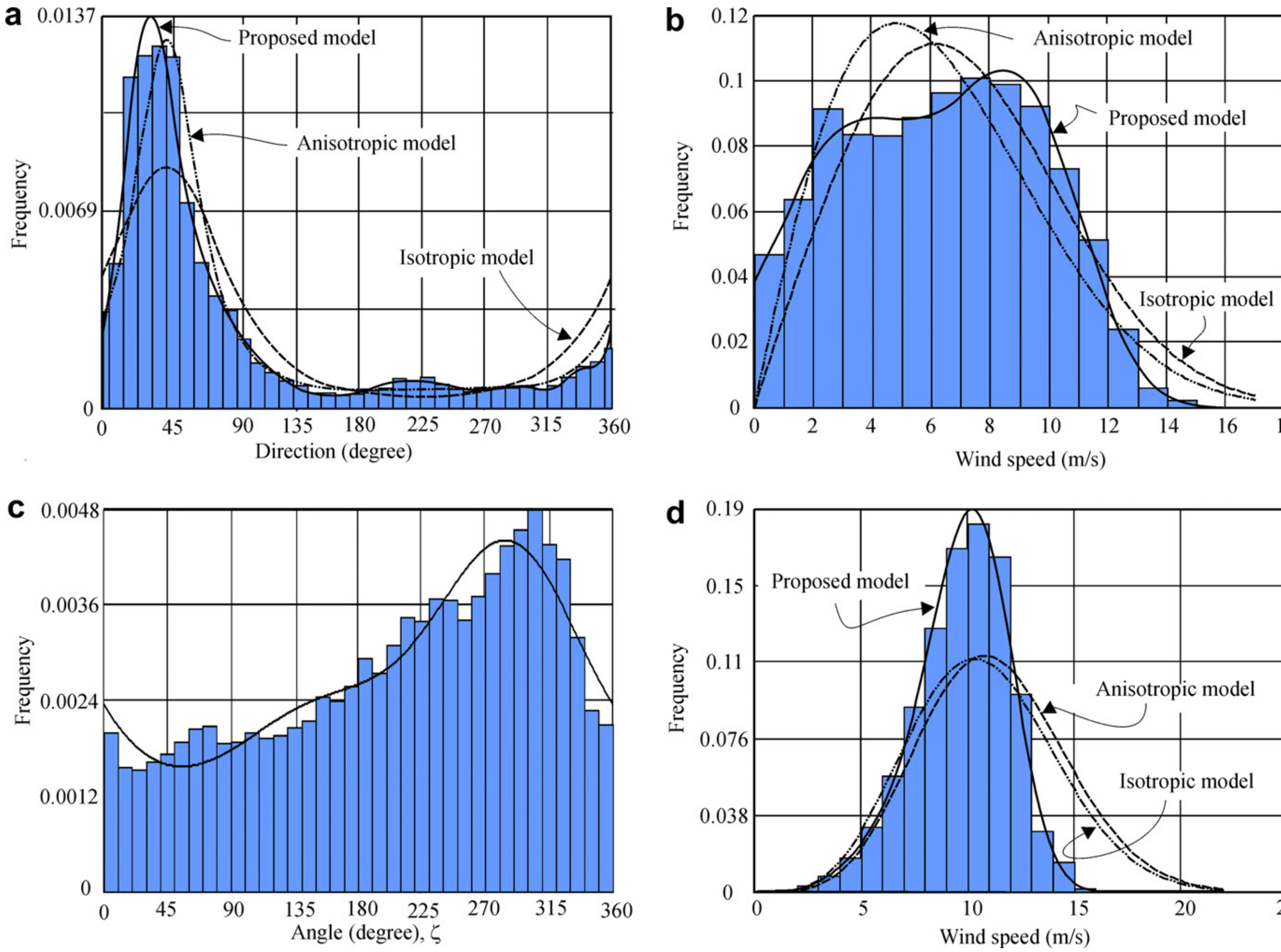

Fig. 5. P. Vargas station: (a) marginal distributions of wind direction, (b) wind speed and (d) wind power probability density function, for the three models analysed. (c) Probability density function of the variable ζ.

two components has provided coefficients of determination, R^2, of the joint probability density functions slightly higher than those obtained when using the uniform distribution.

In order to determine the existing correlation between the hourly sample wind speeds (linear variable) and wind directions (circular variable), we have used the correlation coefficient proposed by Mardia [28]

$$r^2 = \frac{r_{vc}^2 + r_{vs}^2 - 2r_{vc}r_{vs}}{1 - r_{cs}^2} \tag{30}$$

where

$$r_{vc} = \text{corr}(v, \cos\theta); \quad r_{vs} = \text{corr}(v, \sin\theta);$$
$$r_{cs} = \text{corr}(\cos\theta, \sin\theta) \tag{31}$$

As can be seen in Figs. 7a, 8a, 9a and 10a, the values of the coefficients of correlation r^2, at the stations under study, are low.

Fig. 7 shows, for the case of Amagro, the joint probability density function obtained with the proposed model (Fig. 7a) and a graph which represents the theoretical cumulative distribution against the sample cumulative distribution (Fig. 7b), and in which we can see the degree of fit to the sample data of the three bivariable distributions analysed. In Figs. 8–10 the same graphs are shown, but for the stations R. Prieto, P. Vargas and Granadilla, respectively.

It can be seen that, for all the stations analysed, the proposed joint probability density function has a higher degree of fit[13] to the sample data than the bivariable distributions proposed in the specialised literature on wind energy.[14]

The proposed joint probability function can be used to estimate the wind power density per rotor swept area and wind speed. As an example, we can see in Fig. 11a, for Amagro, the wind power density for five wind directions (60°, 79.7°, 90°, 100° and 260°).[15] For the same station, we can see in Fig. 11b the wind power density

[13] The value of R^2 varies between 0 and 1. The higher R^2 is, the greater the fit.

[14] This might be due to the fact that the hypotheses on which the analysed Gaussian models are based are not fully met when applied to the data observed at the weather stations considered.

[15] To represent each curve in the Figure, the considered angle has been put in Eq. (28) and the wind speed $v$has been varied between 0 and $\propto$. We have taken $\rho = 1.225\ \text{kg m}^{-3}$.

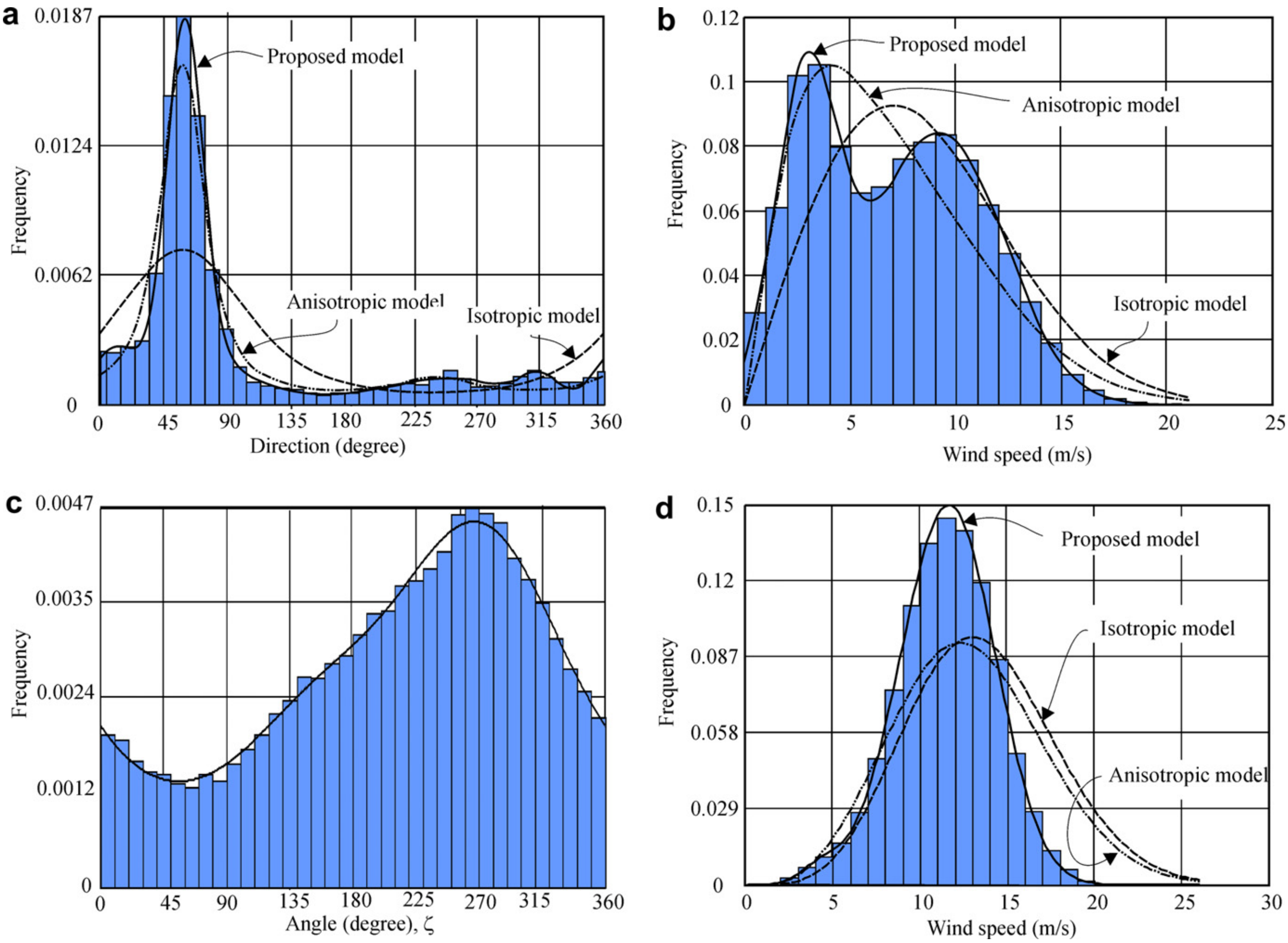

Fig. 6. Granadilla station: (a) marginal distributions of wind direction, (b) wind speed and (d) wind power probability density function, for the three models analysed. (c) Probability density function of the variable ζ.

Table 2
Parameters of the marginal distributions of wind speed

W. station	Normal truncated			Weibull		
	ϕ_1 (m s^{-1})	ϕ_2 (m s^{-1})	ω_0 (–)	α (–)	β (m s^{-1})	$1-\omega_0$ (–)
Amagro	4.272	3.783	0.462	3.471	10.845	0.538
R. Prieto	−0.36	5.504	0.364	3.145	7.816	0.636
P. Vargas	2.791	2.28	0.396	4.008	9.237	0.604
Granadilla	9.193	3.017	0.635	2.272	3.54	0.365

Table 3
Parameters of the marginal distributions of wind direction

Number of components j	Amagro			R. Prieto			P. Vargas			Granadilla		
	μ_j (rad)	κ_j (–)	ω_j (–)	μ_j (rad)	κ_j (–)	ω_j (–)	μ_j (rad)	κ_j (–)	ω_j (–)	μ_j (rad)	κ_j (–)	ω_j (–)
1	0.678	32.501	0.034	0.725	14.792	0.081	0.544	11.412	0.463	0.218	11.149	0.118
2	1.348	19.360	0.621	1.323	15.512	0.519	0.994	4.869	0.297	1.056	18.754	0.601
3	1.339	1.920	0.154	0.403	0.001	0.038	1.776	3.776	0.09	1.683	6.689	0.072
4	3.593	1.029	0.130	0.000	1.140	0.189	3.806	3.105	0.081	3.326	1.031	0.083
5	4.404	19.395	0.044	1.876	13.260	0.125	5.182	5.43	0.041	4.454	2.945	0.085
6	6.283	21.494	0.018	4.401	31.906	0.048	5.905	28.484	0.026	5.473	18.540	0.042

Table 4
Parameters of the probability density function of the variable ζ

Number of components j	Amagro			R. Prieto			P. Vargas			Granadilla		
	μ_j (rad)	κ_j (–)	ω_j (–)	μ_j (rad)	κ_j (–)	ω_j (–)	μ_j (rad)	κ_j (–)	ω_j (–)	μ_j (rad)	κ_j (–)	ω_j (–)
1	3.589	0.8	0.593	4.390	0.620	0.947	2.594	0.687	0.366	2.718	0.968	0.249
2	5.446	1.264	0.407	5.531	4.435	0.053	5.016	1.052	0.634	4.746	0.879	0.751

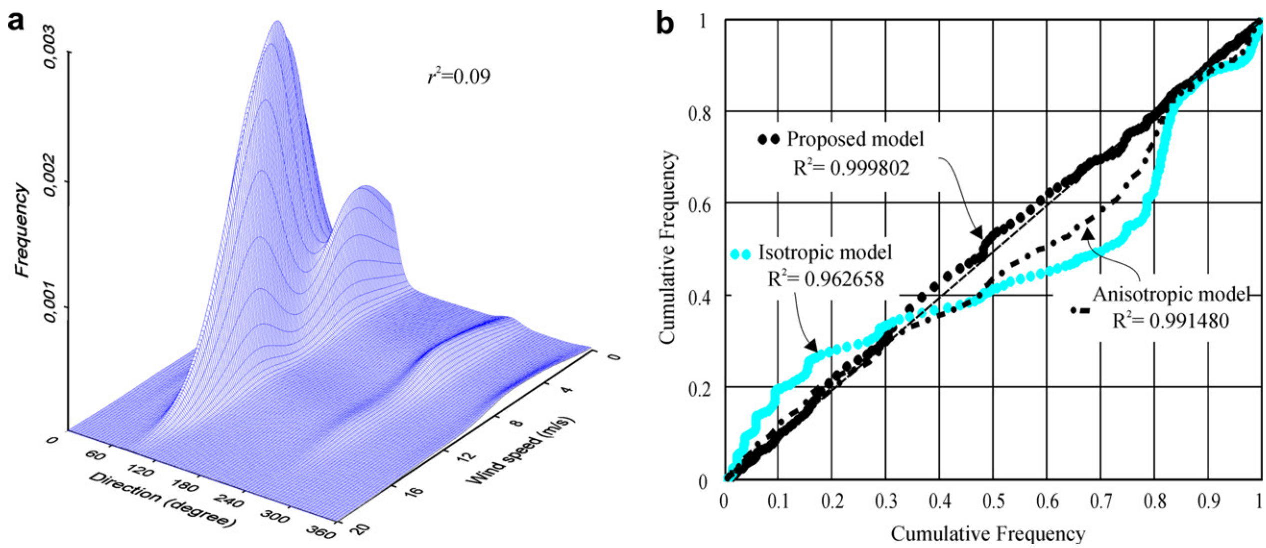

Fig. 7. Amagro station: joint probability density function (a), probability graph (b).

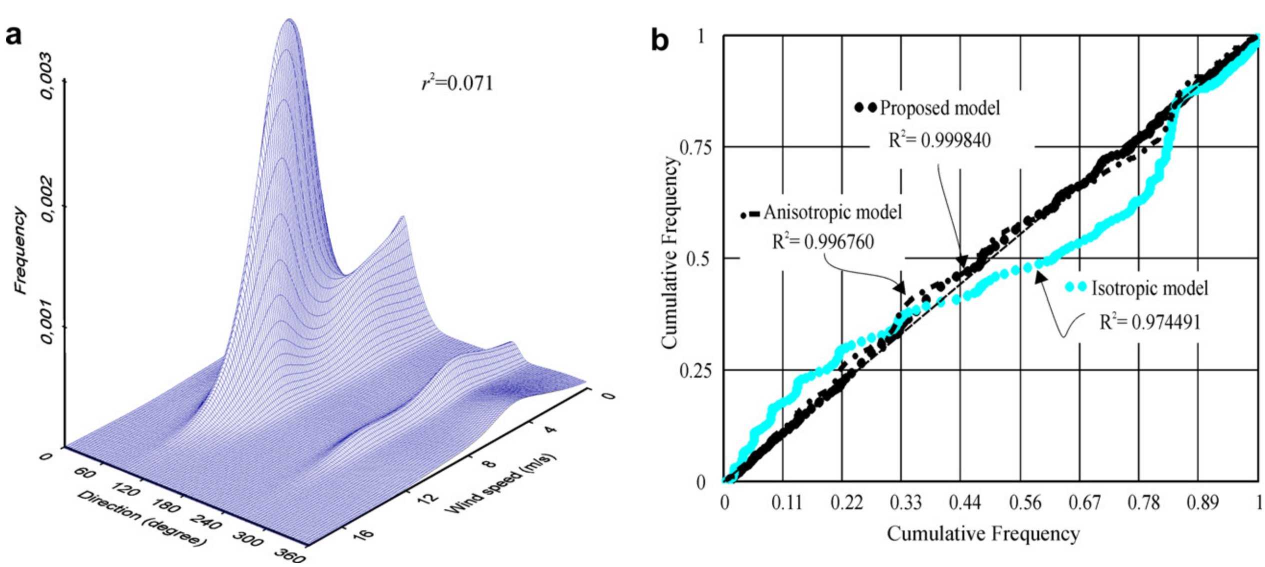

Fig. 8. R. Prieto station: (a) joint probability density function, (b) probability graph.

per rotor swept area and wind direction.[16] In a similar way, for a given wind turbine the obtainable energy can be calculated as a function of the wind direction using Eq. (29).[17]

[16] Each curve was obtained by integrating the wind speed in an interval (0–10 m/s, 10–25 m/s and 0–∞ m/s) in Eq. (28), and representing the wind power density as a function of the variations in wind direction between 0° and 360°. We have taken $\rho = 1.225\ \mathrm{kg\,m^{-3}}$.

[17] Integrating the wind speed between 0 and ∞ in Eq. (29), and the angles in the desired intervals.

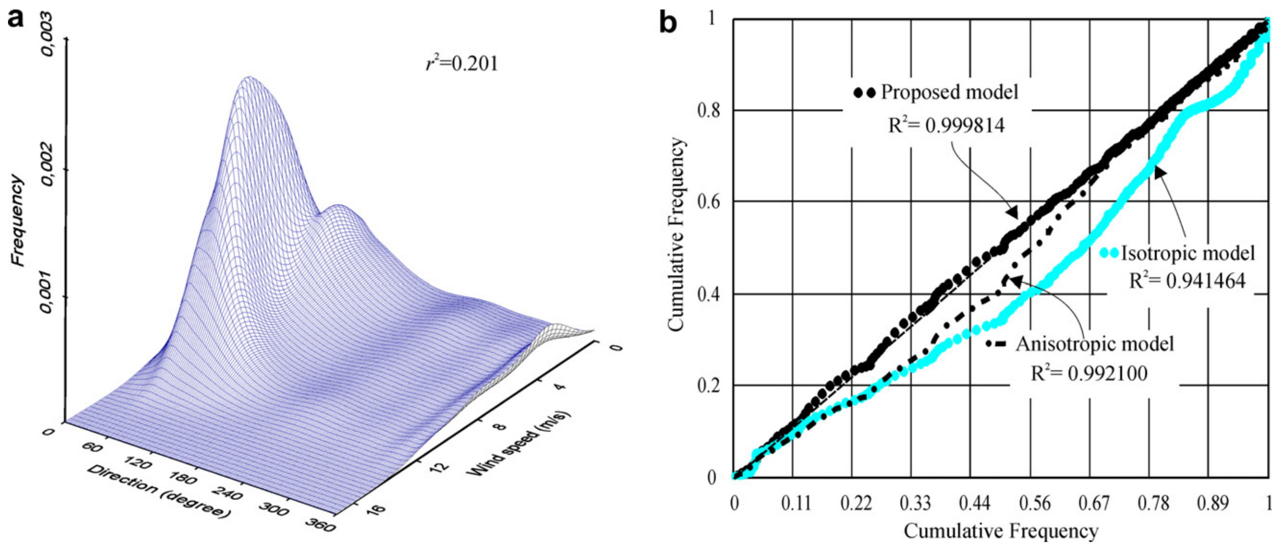

Fig. 9. P. Vargas station: (a) joint probability density function, (b) probability graph.

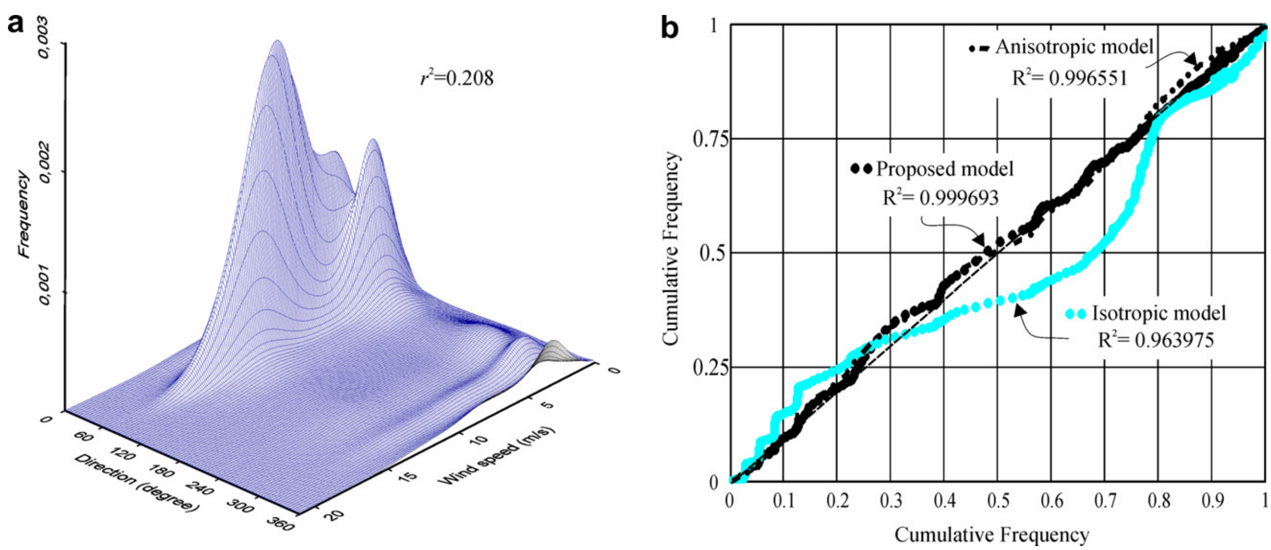

Fig. 10. Granadilla station: (a) joint probability density function, (b) probability graph.

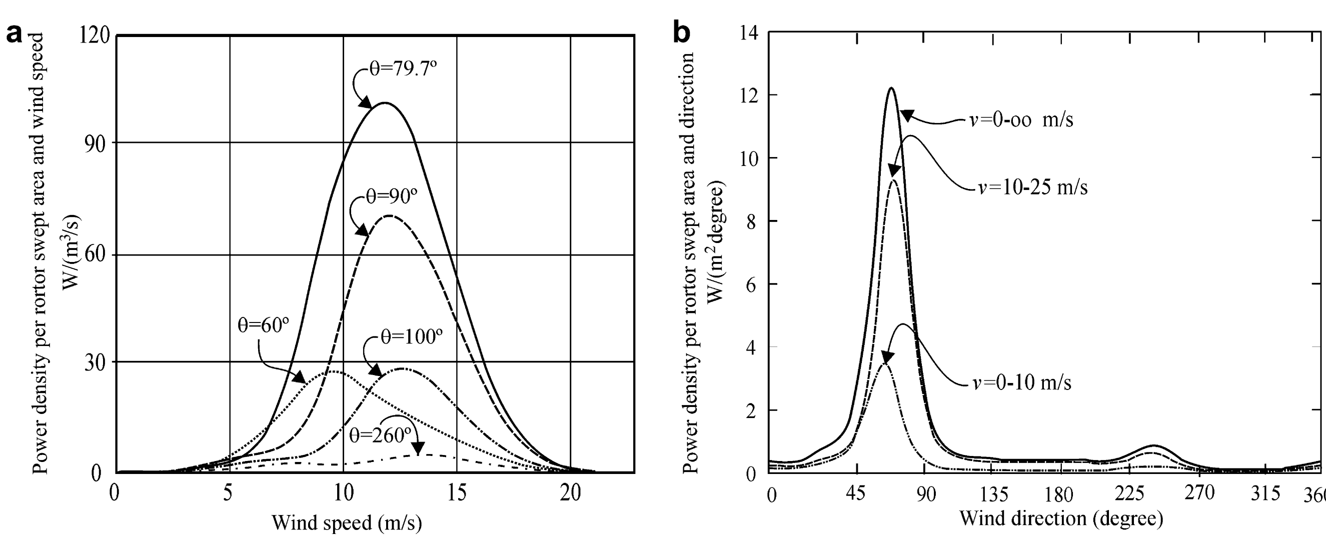

Fig. 11. Amagro station: (a) wind power density per rotor swept area and wind speed, (b) wind power density per rotor swept area and wind direction.

7. Conclusions

The conclusions reached are that the joint probability density function proposed in this paper is very flexible and complete: (a) can represent unimodal, bimodal and bitangential wind speed frequency distributions, (b) takes into account the frequency of null winds, (c) represents the wind direction regimes in zones with several modes or prevailing wind directions, (d) takes into account the correlation between wind speed and direction. It can therefore be used in several tasks involved in the evaluation process of the wind resources available at a potential site. We also conclude that, in the case of the Canary Islands, the proposed model provides better fits in all the cases analysed than those obtained with the models used in the specialised literature on wind energy. It should be pointed out that the analysed stations have been installed in areas suitable for the development of the large scale exploitation of wind energy [26], and have thus avoided terrain of complex topography. We therefore consider that the results of the proposed model are even better when set against those of other models for sites with more complex wind direction histograms.

References

[1] Koeppl GW. Putnam's power from the wind. 2nd ed. New York: Van Nostrand Reinhold; 1982.
[2] Ramírez P, Carta JA. Influence of the data sampling interval in the estimation of the parameters of the Weibull wind speed probability density distribution: a case study. Energy Convers Manage 2005;46:2419–38.
[3] Ramírez P, Carta JA. The use of wind probability distributions derived from the maximum entropy principle in the analysis of wind energy. A case study. Energy Convers Manage 2006;47:2564–77.
[4] Carta JA, Ramírez P. Use of finite mixture distribution models in the analysis of wind energy in the Canarian Archipelago. Energy Convers Manage 2007;48:281–91.
[5] Chadee JC, Sharma C. Wind speed distributions: a new catalogue of defined models. Wind Eng 2001;25:319–37.
[6] Smith OE, 1971. An application of distributions derived from the bivariate normal density function. In: Proceedings of international symposium on probability and statistics in the atmospheric sciences, p. 162–8.
[7] Justus CG. Winds and wind system performance. 1st ed. US: Franklin Institute Press; 1978.
[8] Jones RH. Fitting a circular distribution to a histogram. J Appl Meteorol 1976;15:94–8.
[9] McWilliams B, Newmann MM, Sprevak D. The probability distribution of wind velocity and direction. Wind Eng 1979;3:269–73.
[10] McWilliams B, Sprevak D. The estimation of the parameters of the distribution of wind speed and direction. Wind Eng 1980;4:227–38.
[11] Weber RO. Estimators for the standard deviation of horizontal wind direction. J Appl Meteorol 1997;36:1407–15.
[12] Weber R. Estimator for the standard deviation of wind direction based on moments of the Cartesian components. J Appl Meteorol 1991;30:1341–53.
[13] Manwell JF, McGowan JG, Rogers AL. Wind energy explained. 1st ed. England: John Wiley and Sons, Ltd.; 2002.
[14] García-Rojo R. Algorithm for the estimation of the long-term wind climate at a meteorological mast using a joint probabilistic approach. Wind Eng 2004;28:213–24.
[15] Johnson RA, Wehrly TE. Some angular–linear distributions and related regression models. J Am Statist Associat 1978;73:602–6.
[16] Carta JA, Bueno C, Ramírez P. Statistical modelling of directional wind speeds using mixtures of von Mises distributions: A case study. Energy Convers Manage, doi:10.1016/j.enconman.2007.10.017.
[17] Canavos G. Applied probability statistical methods. 1st ed. New York: Little Brown and Company; 1998.
[18] Draper NR, Smith H. Applied regression analysis. 1st ed. New York: John Wiley and Sons, Inc.; 1998.
[19] Zhang S, Jin J. Computation of special functions. 1st ed. New York: Wiley; 1996.
[20] Hennessey JP. A comparison of the Weibull and Rayleigh distributions for estimating wind power potential. Wind Eng 1978;2:156–64.
[21] Joe H. Multivariate models and dependence concepts. 1st ed. New York: Chapman and Hall; 1997.
[22] Kiryanov D. The Mathcad 2001i handbook. 1st ed. USA: Delmar Thomson Learning; 2002.
[23] Spera DA. Wind turbine technology. 3rd ed. New York: ASME Press; 1995.
[24] Carta JA, Mentado D. A continuous bivariate model for wind power density and wind turbine energy output estimations. Energy Convers Manage 2007;48:420–32.
[25] Bueno C, Carta JA. Technical–economic analysis of wind-powered pumped hydrostorage systems. Part I: Model development. Sol Energy 2005;78:382–95.
[26] Calero R, Carta JA. Action plan for wind energy development in the Canary Islands. Energ Policy 2004;32:1185–97.
[27] Mardia KV, Jupp PE. Directional statistics. 2nd ed. Chichester: Wiley; 2000.
[28] Mardia KV. Linear–circular correlation coefficients and rhythmometry. Biometrika 1976;3:403–5.